Emil Artin in 1927 proposed a conjectural density for the set of primes p for which a given integer g is a primitive root modulo p. However, in 1957 computer calculations by D.H. and E. Lehmer showed unexpected deviations for certain small g. Artin, when informed of this, reconsidered his original statement and came with a correction. The Lehmers, unfortunately, do not seem to have mentioned this in any of their papers. The correction is now sometimes attributed to Heilbronn. It is therefore not without historical interest that the author and H.W. Lenstra unearthened the correspondence between the Lehmers and Artin. This correspondence makes for amusing reading, as there is an interesting clash of mathematical background and outlook involved. Some of the highlights of this correspondence are reprinted and discussed in the first part of this paper.

Assuming the Generalized Riemann Hypothesis it was proved in 1967 by C. Hooley [J. Reine Angew. Math. 225 (1967), 209-220; MR0207630] that the density for Artin's problem exists and equals a rational number times the so called Artin constant $A = \prod_{p \text{ prime}}(1 - \frac{1}{p(p-1)}) \approx 0.3739$. On averaging these densities of g, their average turns out to be A. Thus Artin's constant can be considered to be the density for a generic number g. Many variations of Artin's problem follow the same format. Unfortunately, analytic number theory gives the densities involved in terms of sums involving the Möbius function. It can then be (and usually is) a challenge to derive the actual density in Euler product form. Indeed, aleady Hooley spoke of a 'rather harder' calculation [ibid., p. 219].

H. W. Lenstra, the author and the reviewer [in preparation] have found a way to infer the Euler product in a much more direct way. Also for the first time the correction term is written as a product of local terms. The format of the density is then the generic positive density δ multiplied by $1 + \prod_p E_p$, where the E_p are averages of real character sums and p runs over the primes. Note that $-1 \leq E_p \leq 1$. Thus it has become fairly straightforward to describe when the density vanishes (an odd number of them have to equal -1 and the remaining one have to equal one which happens only under rather restricted conditions on the parameters of the problem involved).

In the second part of this well-written paper the author explains and illustrates the method of the latter three authors in case of Artin's original problem and a variaton where one asks for primes $p \equiv a (\text{mod } f)$ such that, moreover, g is a primitive root modulo p. Hooley's 'rather harder' calculation is rendered almost immediate by this approach. In general it is seen that the character sum method reduces the calculation of the correction factor to a number of fairly mechanical local computations. For the primes in progression variant this is amply demonstrated by comparing the author's calculation with one of the reviewer (Max-Planck-Institut preprint MPI-1998-57, unpublished) that evaluates this density in the classical way.