The function $\Psi(x,y)$ counts the number of integers $n \le x$ that have only prime divisors $\le y$. Relatively good estimates for $\Psi(x,y)$ are required in the analysis of the running time of many integer factoring and discrete logarithm algorithms. Since computing the exact value of $\Psi(x,y)$ is quite prohibitive in practice, one is forced to use an approximation. The most well-known approximation is $\Psi(x,y) \approx x\rho(\log x/\log y)$, where ρ denotes the Dickman-de Bruijn function. Another approximation arises from the work of A. Hildebrand and G. Tenenbaum [Trans. Amer. Math. Soc. 296 (1986), no. 1, 265-290; MR 87f:11066]. Put $\zeta(s,y) = \prod_{p \le y}(1-p^{-s})^{-1}$ and let $\varphi_2(s,y)$ be the derivative of the logarithmic derivative of $\zeta(s,y)$. Let $\alpha(x,y)$ be the unique solution to the equation $\log \zeta(\alpha,y) + \log x = 0$. Then Hildebrand and Tenenbaum's approximation for $\Psi(x,y)$ is $x^s\zeta(s,y)/(s\sqrt{2\pi\varphi_2(s,y)})$, with $s = \alpha(x,y)$. Based on this approximation, the author and S. Hunter presented in [Math. Comp. 66 (1997), no. 220, 1729-1741; MR 98c:11093] a simple algorithm that approximates $\Psi(x,y)$ in $O(y(\log\log x/\log y + 1/\log\log y))$ floating point operations.

In the present paper the author shows how to modify this algorithm to improve its running time to roughly $\sqrt{y}$ floating point operations. The idea is to truncate $\alpha(x,y)$, $\zeta(s,y)$ and $\varphi_2(s,y)$ at $\sqrt{y}$ and approximate the tail using a version of the Prime Number Theorem which assumes the validity of the Riemann Hypothesis. The output of this algorithm is compared in the final section with the output of the earlier, more exact, algorithm.