Let $a > 1$ and p a prime number not dividing a. The least positive k such that $a^k \equiv 1 (\mathrm{mod}\ p)$ we denote by $\mathrm{ord}_p(a)$. Denote the number of primes $p \leq x$ with $(p, a) = 1$ such that respectively $m | \mathrm{ord}_p(a)$, $(\mathrm{ord}_p(a), m) = 1$, $m || \mathrm{ord}_p(a)$, by respectively $N_0(x, m, a)$, $N_1(x, m, a)$, $N_2(x, m, a)$ ($n || \mathrm{ord}_p(a)$ means that for each $q^\alpha || n$ we have $q^\alpha || \mathrm{ord}_p(a)$, where q is a prime). The author, extending his earlier work [Acta Arith. 34 (1977/78), no. 3, 183–196; MR 58#5554a] and [Acta Arith. 43 (1984), no. 2, 177–190; MR 86e:11081], proves estimates of the form

$$N_j(x, m, a) = \alpha_j(m, a) \frac{x}{\log x} + O\left(\frac{(\log \log x)^{r+3}}{(\log x)^2} \right),$$

with $\alpha_j(m, a)$ explicitly given (but complicated), $0 \leq j \leq 2$ and r the number of prime divisors of m. The proof requires no new mathematical ingredients beyond those employed in the quoted earlier work.

The author has been studying the divisibility of the order already since the end of the 1970's and meanwhile obtained very general results. Unfortunately several of the author's papers in this area are regularly overlooked. For example, in K. Wiertelak [Acta Arith. 34 (1977/78), no. 3, 183–196; MR 58#5554a] comparable results with those in R.W.K. Odoni's paper [J. Number Theory 13 (1981), no. 3, 303–319; MR 83a:10098] are achieved, but sofar Odoni's paper is quoted more often in the literature.