Multiple zeta values

Speed talk

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Multiple zeta values

Definition (MZV)

For $s_i \in \mathbb{Z}_{>0}$, a multiple zeta value is

$$\zeta(s_1, s_2, \ldots, s_k) := \sum_{0<n_1<n_2<\cdots<n_k} \frac{1}{n_1^{s_1} n_2^{s_2} \cdots n_k^{s_k}}$$

- Lots of algebraic structure, and proven relations (double shuffle, duality, ...)
- Still many conjectural identities

$$2\zeta(3,3,\underbrace{\{2\}^n}_{\text{2 repeated } n \text{ times}}) - \zeta(3,\{2\}^n,1,2) \stackrel{?}{=} -\zeta(\{2\}^{3+n}) \qquad \text{(Hoffman)}$$

Generalisation of Hoffman's identity

Theorem (C.)

The following identity holds

$$\zeta(\{2\}^a,3,\{2\}^b,3,\{2\}^c)-\zeta(\{2\}^b,3,\{2\}^c,1,2,\{2\}^a)+\\+\zeta(\{2\}^c,1,2,\{2\}^a,1,2,\{2\}^b)\in\zeta(\{2\}^{3+a+b+c})\mathbb{Q}$$

- Proof uses motivic MZV's
- Bigger generalisation; includes new and old conjectures like

$$\sum_{\text{cycle } a_i}\zeta(\{2\}^{a_1},1,\{2\}^{a_2},3,\{2\}^{a_3},1,\{2\}^{a_4},3,\{2\}^{a_5})\\ \overset{?}{=}\zeta(\{2\}^{4+\sum a_i}) \qquad \text{(BBBL)}$$