Warmup (2i, 2ii)

Factorise these into irreducibles in $\mathbb{Q}[x]$

- $f_1(x) = 3x^2 + 2x + 1$
- $f_2(x) = x^8 + 164x^6 + 24x^3 + 2$

Small degree

Recall how 'small' degree polynomials can factor.

- $\deg 2 = \deg 1 \times \deg 1$
- $$\begin{aligned} \deg 3 &= \deg 2 \times \deg 1 \\ &= \deg 1 \times \deg 1 \times \deg 1 \end{aligned}$$

Small degree

- $$\begin{aligned} \text{deg } 4 &= \text{deg } 3 \times \text{deg } 1 \\ &= \text{deg } 2 \times \text{deg } 1 \times \text{deg } 1 \\ &= \text{deg } 1 \times \text{deg } 1 \times \text{deg } 1 \times \text{deg } 1 \\ \\ &= \text{deg } 2 \times \text{deg } 2 \leftarrow \text{Remember this!} \end{aligned}$$

Rational roots

Recall how to find rational roots of $f \in \mathbb{Z}[x]$. Say

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \,,$$

has root $p/q \in \mathbb{Q}$.

- Then p divides a_0,
- And q divides a_n.
- Think about $2x + 3$ if you forget. . .

Question 2i

$$f_1(x) = 3x^2 + 2x + 1$$

- Degree 2
- So irreducible iff no linear factors
- Linear factors $\leftrightarrow$ rational roots
- Rational root candidates: $\pm 1, \pm \frac{1}{3}$
- No roots, so irreducible

Eisenstein

Polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \in \mathbb{Z}[x]\,,$$

and prime $p \in \mathbb{Z}$. If:

- $p \mid a_i$, for $0 \leq i \leq n-1$,
- $p \nmid a_n$, and
- $p^2 \nmid a_0$.

Then $f(x)$ is irreducible in $\mathbb{Q}[x]$.

Question 2ii

$$f_2(x) = x^8 + 164x^6 + 24x^3 + 2$$

- Degree 8 (!)
- No linear factors
- Try Eisenstein?
- Take $p = 2$, conditions hold
- So irreducible