ALGEBRA 2 – MICHAELMAS 2015
TUTORIAL 3

2) Let $R = (\mathbb{Z}/2)[x]$, and let I be the ideal $(x^2+1)_R$ of R.

i) Let $f(x) \in R$. Use the division algorithm to prove that $f(x) - (ax+b) \in I$ for some $ax+b \in R$. Are a and b unique? Hence list the elements of $(\mathbb{Z}/2)[x]/I$.

ii) Check the following computations in $(\mathbb{Z}/2)[x]/I$

- $(\overline{1}+I)+((x+\overline{1})+I) = x+I$,
- $(x+I)\cdot(x+I) = \overline{1}+I$.

iii) Complete the addition and multiplication tables for $(\mathbb{Z}/2)[x]/I$.

iv) **Extra:** Is $(\mathbb{Z}/2)[x]/I$ a field?