ALGEBRA 2 – MICHAELMAS 2015
TUTORIAL 3

Remark: In question 2 we were able to find a fixed degree (in that case degree 1) representative for every equivalence class because we worked with polynomials over a field (in that case $\mathbb{Z}/2$).

If we don't work with polynomials over a field, this might not be possible. The division algorithm won't be applicable. There may be quotients rings were equivalence classes need arbitrarily high degree representatives.

Example: Let's look at $\mathbb{Z}[x]/I$, with $I = (2x+1)_{\mathbb{Z}[x]}$. No fixed degree is enough to get representatives for every equivalence class(!)

If we could reduce always to $(a_n x^n + \cdots + a_0) + I$, for some fixed n, then $x^{n+1} + I$ would have to be equivalent to one of these. But I claim $x^{n+1} + I \neq (a_n x^n + \cdots + a_0) + I$. Indeed

$$2x + 1 \nmid x^{n+1} - (a_n x^n + \cdots + a_0)\,.$$

This is because any multiple of $2x+1$ has *even* leading coefficient. This means $x^{n+1} - (a_n x^n + \cdots + a_0) \notin I$, and the equivalence classes are not equal.