Durham University Learning and Teaching Award

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24 October 2014

Integrity sheet

I declare that all material in this portfolio is my own work except where there is clear acknowledgement and reference to the work of others. I give permission for my assessment work to be reproduced and submitted to other academic staff for the purposes of assessment and to be copied, submitted to and retained by the University's plagiarism detection software provider for the purposes of electronic checking of plagiarism.

Signature:	....
Date:	24 October 2014
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I grant permission for an anonymised version of my submission to made available to future students

Yes: ✓ No:

Contents

Checklist of Learning Outcomes

This portfolio provides evidence of my:

K:	**Tick**	**Page**
Knowledge and understanding of:		
K1 relevant subject material in my field;	✓	1, 7
K2 appropriate methods for teaching and supporting learning in my field;	✓	3–4, 6, 8
K3 how students learn, both generally and in their subjects;	✓	3, 5
K4 the use of appropriate technologies to support their learning;	✓	3, 6, 8
K5 how to evaluate the effectiveness of my teaching;	✓	5, 10
K6 the implications of quality assurance and enhancement for my professional practice.		

A:	**Tick**	**Page**
Skills and abilities to undertake at least four activities drawn from the list below, which must include A5 and A6:		
A1 design and plan learning activities;		
A2 teach appropriately and to support student learning;	✓	3–4
A3 assess student learning and give feedback;	✓	5–6
A4 develop effective learning environments and support and guide students;		
A5 integrate scholarship, research and professional activities with teaching and supporting learning;	✓	7–8
A6 evaluate my practice and plan continuing professional development.	✓	10–11

V:	**Tick**	**Page**
Appropriate values of:		
V1 respect for individual learners and diverse learning communities;	✓	1, 2, 3
V2 promote participation in higher education and equality of opportunity for learners;	✓	1, 2, 3, 6
V3 use evidence informed approaches and the outcomes from research, scholarship and continuing professional development;	✓	7–8, 10–11
V4 acknowledge the wider context in which higher education operates and recognise the implications for professional practice.	✓	3, 7, 10

1 Statement of teaching context

This portfolio covers (primarily) my teaching role in the Department of Mathematical Sciences during the academic year 2013–2014. In this section I provide an overview of my teaching experiences thus far, with the aim of providing a context for the remainder of the portfolio.

I began my PhD in the Department of Mathematical Sciences at Durham in October 2012, after successfully completing a MMath degree there as an undergraduate.

During the fourth year of my undergraduate, I took up the opportunity to mark homework for three tutorial groups of students on the second year Algebra II module, a module I had taken myself in a previous year. In preparation for marking I attended the departments New Marker session, which familiarised me with the university and department guidelines for marking assignments, and the logistics of marking in the department. This means that I was already very familiar with the marking and assessment procedures at Durham from both points of view, and was very familiar with the course material.

During the first year of my PhD I continued marking. This time I assisting with the marking for the third/fourth year Number Theory III/IV module, another module I had taken as an undergraduate. In both of these cases, the only interaction I had with the students was via the written feedback I provided on their assignments. And indirectly via feedback given to the lecturers. All in all, this forced me to consider carefully how to best structure each student's feedback to communicate clearly good and bad points of their solutions.

During the fourth year of my undergraduate I also took on what might be described as an informal teaching role with a small group of peers, as we studied the fourth year extra reading material. This was more rigidly structured than just a discussion amongst friends; comments from them about how well I helped them to understand the material suggested I might have an aptitude for teaching.

Upon starting my PhD I attended a faculty wide Teaching Induction Session concerning small group teaching in the sciences. The session focused on quickly getting us up to speed with university expectations and requirements for teaching and marking, and giving us the basic skills to handle a class of students. The session was compulsory for all new students who would take on any teaching or marking duties over the course of their PhD.

My first formal teaching experience came in the second year of my PhD, after I decided to sign up for three Analysis 1 tutorials, and three Core A computer classes. The Maths department typically does not offer teaching roles to first year students; when the sheet for 2013–2014 tutorials came round, I quickly signed up.

The Core A computer class was one session in the first week of Michaelmas term, where new Maths students would be given an introduction to the Maple computer algebra system. The structure of the session was very rigidly prescribed by the department: students have a worksheet to lead them through a series of examples of using computer algebra systems in the context of A-Level questions. Primarily I was there to provide help to students who, for whatever reason, were struggling to make the examples work. However I anticipated that some students would have experience with coding/programming-style activities and would finish the worksheet very quickly. For these students, I thought up some short challenges they could test their skills with.

At the end of the 2013–2014 academic year, I volunteered to run one session of a LaTeX drop-in class. The class was organised by the department for second and third year students doing a Maths project in the 2014–2015 academic year, as they would be (virtually) required to write their report using LaTeX. I was given no particular directions for how the session was to be run, save to be prepared for possible questions ranging from installing and configuring a TeX distribution; to creating, structuring and formatting simple documents; through to complex customisation of commands. In case all else failed, and some students just turned up wanting to get their hands dirty and 'play' with LaTeX, I put together my own short worksheet illustrating some of the basic commands for structuring documents, formatting text, and writing equations, which I ended up using.

My main source of teaching experience this year has been from Analysis 1 tutorials. In preparation for taking on tutorials, I attended the departments New Tutor session which gave us an overview of the university and department expectations for tutorials, and the logistics of tutorials in the department. This session was follows by a lively discussion between old and new tutors. It covered the pros and cons of various tutorial styles, the successful techniques and the potential pitfalls, and the various tips and tricks they'd gleaned over the years of tutoring.

Tutorials in the mathematics department ideally should take the form of a small group discussion (approximately 10–15 people) drawing on material from the content of previous lectures, the exercises suggested in advance by the lecturer, the problematic points of recent homework. Analysis 1 tutorials came with the expectation that I would mark my students homework, and give them feedback. Naturally, this role as tutor comes with responsibilities outside of the one hour tutorial slot each week. In order to effectively guide students through the problematic points of questions, I must familiarise myself with the questions set and their solutions. Moreover, I must try to anticipate where problems are likely to occur, and develop suitable explanations and examples to overcome them. I also remain available outside of tutorials to provide immediate or individual help to students via email or in person.

2 Teach appropriately and to support student learning

Maslow [12] identifies a hierarchy of needs, from basic to increasingly sophisticated, which govern human motivation. At the basic level are *physiological* needs required for survival. Then comes the needs for *safety*, *belonging*, and *esteem*. These are prerequisites for the *self-actualisation* need encompassing problem solving, and creativity; it is this we want to engage in tutorials when tackling the questions.

In Analysis 1, I am teaching mainly first year students who have just started university. To provide them a safe and welcoming environment which will allow us to reach self-actualisation, I started the first tutorial with a gentle ice-breaker, inspired by 'Getting to Know You' from Strawson et al. [19, p. 15]. I introduced myself to the class, giving my name, office, and email address. I then invited everyone to introduce themselves with their name, college, and home-town. I am not fond of ice-breakers which ask for favourite such-and-such, or an 'interesting' fact about yourself; I feel they induce unnecessary pressure and feelings of being judged making them poor ice-breakers. For this reason, I chose those questions, knowing they would elicit un-controversial answers. This allowed me to put faces to names, and over the coming weeks I would learn all students' names by reinforcing this association when returning homework, and speaking to students in class.

After the ice-breaker we discussed homework arrangements. Due to a quirk of how the lecturer organised the first homework, I already had a set of marked work to hand back. I was fully aware the first deadline I set was unnecessarily short, for which I apologised. After discussing the options with students, we agreed a new deadline acceptable to everyone – hand in homework the Monday one week after it's assigned. They would have sufficient time to do it, I would have time to mark and return it by their next tutorial. This was akin to a 'Contract' [19, p. 19], although I never set up the agreement in such a formalised framework.

After returning the homework and covering any problems, see Section 3, we moved to the main part of the tutorial. During the maths department's new tutor training session, more experienced tutors shared their experiences with tutorials. The discussion focused on advantages and disadvantages of different tutorial styles, and how students responded to them. After much thought, including reflecting on my own tutorial experiences whilst a student, and weighing up what I thought constitute good mathematical and transferable skills to develop, I decided to try the 'small groups' style of tutorials initially. This is broadly similar to the 'Syndicate Sub-groups' technique described by Strawson et al. [19, p. 47], with slight modifications.

This is a tutorial style I didn't experience much at undergraduate level. Only one tutor ever tried it out, and only for one tutorial. Because this happened randomly in the middle of the term, I don't think the tutorial got the positive reaction it should have. I wasn't going to let this one negative experience dissuade me initially. I found the idea intriguing.

As McLone [14, p. 1] explains, 'mathematical mastery can only be achieved through *doing* mathematics'. Doing mathematics is the key distinction between problems-classes[1] and tutorials. I did not want to turn my tutorials into mini-problems-classes with students just obediently copying down model solutions. Undoubtedly, this would lead only to a superficial surface-level understanding; the active involvement required to promote deep learning wherein students build connections and meaning would be minimal [3, p. 89]. This would leave students struggling to complete non-routine questions which deviate slightly from that they've been 'trained' for. I also suggest it is somewhat redundant because students can see such solutions at the end of term anyway.

[1] In the mathematics department these are classes where the *lecturer* solves problems

With this in mind, the 'small groups' style seemed the natural candidate, with many advantages. By breaking the class into smaller groups, individuals are more likely to contribute because they are working in smaller, safer, environments. This would actively promote discussion of the material within the groups and the class as a whole and would lead to a deeper and fuller understanding because '[students] can only learn effectively if they attempt to explain what they have encountered to their fellow students' [13, p. 8].

I initially began this part of the tutorial by instructing everyone to break into three equal groups, to work through the questions, and that I would circulate checking solutions and offering advice. At the end, we'd reconvene as a class to discuss the questions. This number of groups fits perfectly across the three boards of the room, yet gives groups small enough to be manageable.

To maximise the amount of discussion and interaction, other tutors advised the following. In each group there is a designated *scribe* who's job is writing the solution as directed by the others; the others collaborate to find a solution and talk the scribe through it. After each question, or part, the job of scribe passes on. I asked the students to try this from the first tutorial onwards, but the setup is quite artificial and difficult to maintain even during one tutorial without continual reinforcement. Eventually I allowed this aspect of the setup to disappear, and the students degenerated to normal group work.

Throughout the year I did not monitor the groupings as rigorously as possible. After a few tutorials the same groups would form repeatedly. Possibly this is a good thing, it shows students in each group are comfortable with each other and can work productively together. On the other hand, there is the danger that groups become complacent and set in their ways, with the same individuals doing disproportionate amounts of work.

On reflection, if I run this style of tutorial again, I would more frequently rearrange the groups to guard against such stagnation. I would also try more actively to retain the scribe/solvers aspect, although I would not force this unnecessarily. I've had mostly positive feedback with this style, see Appendix B, more so in the second term after the routine became established. This gives me the confidence to consider running future tutorials like this. I always intend to discuss the questions together towards the end of the tutorial to ensure everyone is happy. However, there were a number of occasions when we ran out of time and had to condense this class discussion. These are precisely the times when feeding back to the class would be most beneficial as it suggests there were difficulties with the problems. In order to reserve time to discuss difficult problems at the end, I will endeavour to cut off the group-work with at least several minutes to spare. I am also inclined to suggest students move to later questions if necessary, so they have a change to see all the material possible.

3 Assess student learning and give feedback

Throughout the literature various purposes for assessment are discussed, by authors including Brown, Bull, and Pendlebury [1, pp. 10–12], and Butcher, Davies, and Highton [3, pp. 94–96]. Freeman and Lewis [5, pp. 10–12] fit these various purposes into five categories: *to select* for future undertakings by providing evidence of suitability; *to certificate* by confirming that the student is competent or has reached a particular standard; *to describe* what the student has learned or can do; *to assist learning* by motivating the students, giving them opportunities to practice, and giving feedback to identity strengths and weaknesses; and finally *to improve teaching* by giving teachers opportunities to the effectiveness of their teaching and make appropriate adjustments.

Within Analysis 1, all summative assessment comes in the form of a three hour end-of-year exam. The remaining assessment consists of formative weekly assignments, and a collection exam at the start of Epiphany term. As a tutor, I am responsible only for the formative parts of the assessment. These assessments, therefore, are primarily designed to assist learning, and give a continuous check on student progress. I can also use them to benchmark the effectiveness of my teaching by identifying areas of my tutorials which need to be clarified and better explained. Here I use incidental feedback to evaluate my teaching.

Various authors identify any number of principles which must hold for good feedback. Consider Nicol and Macfarlane-Dick [17] and Chickering and Gamson [4] each give seven (somewhat distinct) principles. I find the summary McTighe and O'Connor [15, p. 3] give of the discussion by Wiggins [21, p. 43–70] on feedback an agreeable compromise between the number and specificity of principles. They summarise 'to serve learning, feedback must meet four criteria: it must be timely, specific, understandable to the receiver, and formed to allow for self-adjustment on the student's part.'.

Certainly, the feedback I provide is timely. As mentioned, I agreed a Monday deadline for each weekly homework. I then always succeeded in marking each group and returning it by their next tutorial, Friday being the latest. This gives me a consistent turn-around time of under a week, and allows the students to act on feedback before submitting future assignments.

I endeavour to provide specific and understandable feedback. To ensure consistency between students, my marking and grading follows university and departmental guidelines, grading on a scale A–E where A indicates essentially correct and complete work, through to D and E respectively show little or no understanding with significant work omitted. Grades D and E are deemed unsatisfactory. I try to mark fairly, but as strictly as possible, so I can identify chronic problems early, and prevent students unduly suffering with them later.

If they just have a numerical slip, but an otherwise correct solution, all I need to do is indicate where the numerical error first appears. If the numerical error stems from a more obscure problem, say misremembering an important property of a function (thinking $\log(a) + \log(b) = \log(a + b)$, not $= \log(ab)$), then I provide a reminder of the corrected property and a specific counterexample to reinforce why the misremembered version can't be right.

More serious problems occur when students attempt to employ fallacious or poorly explained logic, and so have an incorrect solution even though they might reach a correct final line. Faulty logic is often a subtle mistake to diagnose, and can be a very difficult mistake to explain to the student.

Instances of this problem occur in many ways: commonly, when asked to prove a statement, students will take the purported statement, and deduce from it until they reach a true conclusion, then declare it proven. This exposes their grievous misunderstanding of logic. To counter such misunderstandings I devise the simplest examples possible which illustrate clearly and powerful such a thing cannot be true. I might say to the above: 'The implications are backwards. This couldn't work otherwise $1 = 2$, since $1 = 2 \implies 1 \times 0 = 2 \times 0 \implies 0 = 0$, and the latter is true.'.

Since the first tutorial and homework, I have encouraged students to explain their arguments clearly and present readable work. Communicating one's thoughts clearly is of paramount importance in mathematics. This has not met with total success; it is still necessary to emphasis on many scripts how to better phrase particular aspects of arguments. Currently I make use of a $+/-$ grade to penalise poor explanations and structuring, and other small mistakes, but I definitely need to consider more targeted action in future to improve this situation.

On a few occasions students have asked me for clarification about their feedback. This suggest I should work harder in some cases to craft understandable feedback, but at the very least shows that some students *are* reading their feedback.

At the start of each tutorial, I return the most recently marked batch of homework to students personally. As this takes a minute or two, this naturally demarcates a short period at the start of the class where students can read through their feedback distraction-free. This gives students the opportunity to immediately reflect on it and formulate any questions they may have about it.

After this, I dedicate the first 5–10 minutes of tutorials to providing verbal and written feedback on the homework to the entire group. This should make it very clear to the group that they are receiving feedback, and it allows me to be more certain everyone has acknowledged it.

For example, if any particular homework question has caused great difficulty for the entire class, I will present a model solution to it on the board, explaining my thinking at every step. This is a tactic Mason [13, p. 146] recommends. Being talked through a solution can be much more beneficial than just reading one; the student can get clarification immediately, and can hear the informal thoughts which don't make it to paper.

Another tactic Mason [13, p. 10 and p. 147] suggests is collecting lists of common errors from homework, providing these to students and requesting explanations of why they are wrong from the group. Often I find myself doing this automatically, although I usually explain the error to the class myself. I recognise prompting the group to explain the error is beneficial for many reasons, notably it encourages group discussion from the very start and it keeps the 'better' students engaged during what could otherwise be an irrelevant part of the tutorial. This is something I will endeavour to do more often in future.

During this I encourage students to voice any problems they have with the questions, homework or course in general. Responses to the questionnaire in Appendix B clearly show students do appreciate this approach to providing feedback the start of tutorials. I am, however, aware of the continual temptation to dwell on homework minutia for too long. Whilst I try to limit the discussion to the first 5–10 minutes, I have on a few occasions taken significantly longer than intended.

4 Integrate scholarship, research and professional activities with teaching and supporting learning

Jenkins and Healey [9, pp. 20–22] identify four main linkages between teaching and research in the four quadrants of a plane where one axis measures the emphasis, either on research content or research processes, and the other axis measures student involvement, as participants or as audience. This leads to the notion of teaching which is research-*led* by incorporating research findings into the curriculum; research-*oriented* by incorporating research methods; research-*based* by designing the curriculum around inquiry-based activities; or research-*informed* by incorporating pedagogical research into the design and delivery of teaching.

As explained by Burn, Appleby, and Maher [2, p. 102], 'first year [mathematics] courses generally consist of things that have *always* been there, things *we* know about, things we feel a student *ought* to know, and things that they will *need* to do [optional] courses later on.'. This is reasonably true of the Analysis 1 module at Durham. The mathematics used to rigorously construct infinitesimal calculus, namely the ε-N and ε-δ definition of limits, dates to the early 1800's, and is very well established and understood. The fact that these ideas are so foundational to the subject, and so deeply rooted in centuries of established mathematics means they are very difficult to relate to any modern research questions.

For this reason, I tend to adopt the research-informed approach to teaching, incorporating pedagogical research into the design and delivery of my tutorials. One particular idea from such research that I have found intriguing is that of a 'threshold concept', as identified by Meyer and Land [16]. They compare a threshold concept to 'a portal, opening up a new and previously inaccessible way of thinking about something.', and observe '[a threshold concept] represents a transformed way of understanding, or interpreting, or viewing something without which the learn cannot progress.' [16, p. 412].

They identify the following likely characteristics of a threshold concept [16, pp. 415-416]. A threshold concept is: *transformative* in that it engenders a significant shift in view point on the subject once understood; *irreversible* in that once acquired is will not be forgotten, or can only be unlearned with considerable effort; *integrative* in that is exposes previously hidden connections in the subject; *bounded* in that the conceptual space will have frontiers with thresholds into new areas; and troublesome. Perkins [18] discusses several reasons why knowledge may be troublesome, including its inherent conceptual difficulty or its 'foreign-ness', coming from perspective which conflicts with one's own.

Meyer and Land [16, p. 413] themselves put forward the concept of a limit in mathematics as an example of a threshold concept, calling them a 'gateway to mathematical analysis'. The limits they discuss are restricted to limits of *functions* at finite points, which the Analysis 1 students encounter in the second term; I would like to discuss some aspects of limits of *sequences* at infinity which make them pertinent example of a threshold concept in addition to previous.

Firstly, the ε-N definition of a limit[2] represents one of the first encounters students at Durham have to university level definition-theorem-proof style mathematics. At this stage students' don't appreciate the necessity of having precise definitions for what seem like very intuitive ideas. Essentially $\lim_{n\to\infty} x_n = L$ can be read naïvely as: as n gets large x_n gets close to L. This exemplifies the troublesome aspect of a threshold concept due to the inherent difficulty of the abstract definition, but also due to the foreign-ness of the necessity such definitions. Following on from the lectures, I attempted to emphasis the necessity of such definitions by drawing out what large and close to really mean in the naïve reading. Dealing with questions on whether such

[2] $\lim_{n\to\infty} x_n = L$ by definition means $\forall \varepsilon > 0\, \exists N$ such that $n \geq N \implies |x_n - L| < \varepsilon$

effort is necessary when the results are intuitive and obvious, for example the limit of a sum is the sum of the limits, gave me the opportunity to discuss with the students the nature of proof in mathematics, what mathematicians mean by obvious, and why mathematicians can't solely rely on their intuition as it can often be wrong.

Certainly, understanding limits is both a transformative process, and essentially irreversible. At their most basic level limits describe what happens as processes are allowed to continue 'to infinity'. Dealing directly with infinity is a notoriously delicate proposition, as any school child knows ('What's infinity minus infinity?') [20]. Once it is fully understood that something intelligible can only really be said about processes 'at infinity' by considering their behaviour at large finite values, this become the default way to think about such questions. Within Analysis 1 this is continually reinforced as questions about convergence of series and integrals fundamentally rely on such ideas. Moreover, this illustrates the integrative aspect of limits when they are acting as a 'gateway to analysis' and illuminate connections within the subject.

Once aspect of limits of sequences and series that might be considered a threshold concept in its own right involves using these definitions to prove results in concrete examples. When dealing with specific examples, one must develop some amount of intuition to guide one to suitable choices and simplifications throughout the proof. In this regard, I found the lack of questions on the module problem dealing with specific examples of these proofs something of an oversight, especially considering how frequently such questions appears in past exams. Once I'd identified this topic as a candidate threshold concept, I put together a handout illustrating via example some techniques and strategies for handling these proofs, and provided a number of problems for the students to try on their own. See Appendix D. I hoped that these resources would provide effective explanations to the students.

Overall, students responded positively to the handouts, and explanations I provided about limits, see Appendix B, Appendix C. However still more could be done to assist with teaching these threshold concepts. During the DULTA seminars I discovered the *Applet ShOp* [10], a suite of interactive Java applets designed to illustrate various advanced mathematical concepts. One applet provides an interactive demonstration of limits and sequences, allowing the user to experiment with various values of ε and N and see how the definition holds in each case. It has been recognised that the use of interactivity can enhance learning [7]. I feel the interactivity provided by the applet, along with the added precision of computer created diagrams, would have been a valuable tool when I was discussing limits with my students. For this reason it is something I intend to make use of in future.

Initially I would plot a simple sequence, $x_n = \frac{1}{n}$, and ask the class what the limit is. With an incorrect answer I could demonstrate shrinking ε eventually makes the definition fail. Whereas for a correct answer I would use the applet to graphically find N for various ε. I would repeat this with further series, as in Figure 1, in conjunction with constructing the formal proof of the limits.

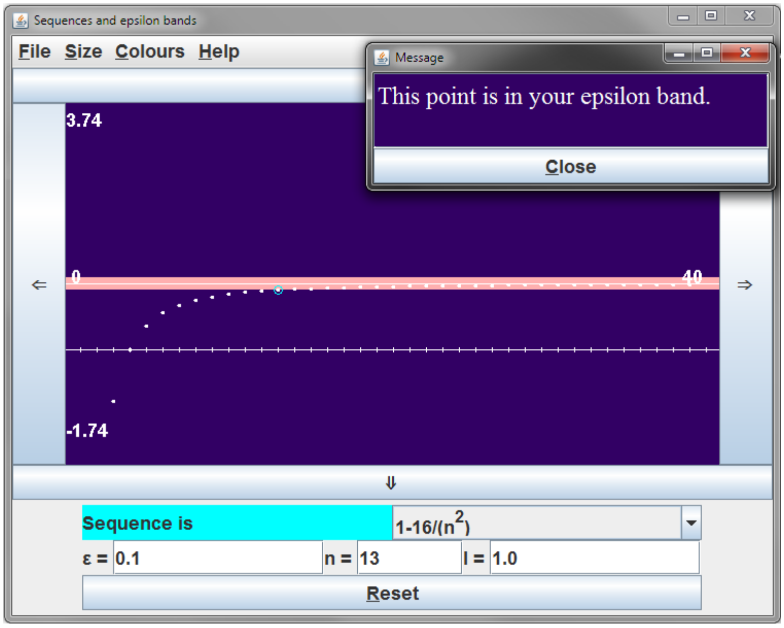

Figure 1: A typical example of the sequences applet in use. The sequence is $x_n = 1 - \frac{16}{n^2}$, and the (correct) candidate limit being used is $L = 1$. The ε-neighbourhood of L is shaded red, where $\varepsilon = \frac{1}{10}$ has been chosen. The term index $n = 13$ is highlighted, tested and reported to lie within this ε-neighbourhood of L.

5 Evaluate my practice and plan continuing professional development

Kahn and Walsh [11, p. 46] define evaluation as 'an ongoing process that can help us to identify and illuminate issues within our teaching practice and alert us to any potential barriers to effective student learning. It allows us to build upon the positive aspects of our practice and address any negative areas.'. Hounsell [8, p. 200–202] identifies three widely-recognised principal sources of feedback for the evaluation process: *feedback from students* via questionnaires, emails, staff-student committees; *feedback from colleagues* via peer observation and collaborative comment; and *self-generated feedback* such as self-evaluation questionnaires and teaching logs. Here Hounsell [8] also identifies a fourth source of feedback, so-called *incidental feedback*, which is 'often underexploited or goes unnoticed'. This feedback is 'to be found in the everyday routines of university teaching and course administration' and includes information such as attendance levels, drop-out rates, patterns of distribution of marks or grades. (This illustrates the use of assessment to improve teaching.)

Within the Mathematics department, student feedback is gathered in the latter half of each term. Students are given a questionnaire to fill out for each module. There are specific questions about: content and delivery, course delivery and communication, work and feedback, resources, tutorials, and the course overall. The relevant parts of the questionnaire for my teaching practice are the questions on work and feedback, and tutorials. Each question is followed by a free response field for any additional comments they wish to make, or any specific points they wish to raise, about these topics. The numerical results, and all free text responses, from Michaelmas 2013 and Epiphany 2014 are reproduced in Appendix B.

Responses to the tutorial question have been mostly positive; between the two terms my average score increased from 77% to 83%. A number of comments in the first term suggest I was a bit nervous, hesitant and unsure when first teaching. There are no comments to this effect in the second term, I feel this is because after the first few weeks of teaching I grew more comfortable with the process, and developed more of a familiarity with the students.

One noticeable feature of the feedback is that in each term one student rates the tutorials negatively, and comments that they 'dislike the setup of the tutorial as I feel little guidance is given when I am finding a question or concept tricky and thereofe [sic] it is difficult to make progress.'. I am very conscious of the fact that the tutorial style I employed for most of the year will not be to everyone's taste; it is unlikely any one style will satisfy everyone. But in future I will make an effort to explore a number of different styles with the students, so that one person is not stuck with an unhelpful tutorial style for the year.

One, quickly apparent, disadvantage with relying on departmental questionnaires as the sole means of acquiring feedback on my tutorials is the lack of any detailed information on the many different aspects of my teaching. Currently I get nothing more than an average overview, and this does not allow me to focus improvements at one specific problem. The delay in receiving the results, combined with their infrequency, means I have few opportunities to identify and implement improvements. Instant questionnaires at the end of tutorials, as described by Gibbs and Habeshaw [6, p. 206], could remedy this by providing frequency feedback on specific problems.

As Hounsell [8] mentions, student feedback does not just have to take the form of student questionnaires. Over the course of the year a number of students have contacted me by email outside of the tutorials, and requested help with particular topics or questions. Some of these students followed up my responses with emails indicating their appreciation for my support and help. I reproduce these in Appendix C. I view these responses as confirmation that I am successfully supporting these students' learning.

The department also requires tutors to undergo peer review; one tutorial is to be observed by another member of the department who is at approximately the same level. The department suggests that after the session observer and observee should exchange ideas and reflect on whatever lessons emerge.

One of my tutorials in Michaelmas term was observed by a member of teaching staff (rather than a fellow PhD student). Those comments are reproduced in Appendix A. My observer found no major problems with my tutorial, and in fact was pleasantly surprised with how well suited my tutorial style (of small groups working at the boards) was to tackling the Analysis 1 questions. They even considered trying it out with their own tutorial groups. Later on in Easter term, I asked a fellow PhD student if they would observe my tutorial and give me some feedback; I felt that having a peer review from someone at the same stage would allow them to better empathise, and identify problems.

Throughout the year I have also had numerous lengthy discussions with fellow tutors. Some of the discussion has revolved around the advantages and disadvantages of various teaching styles, sharing various general tips and suggestions for teaching. At other times, I have discussed with Analysis 1 tutors how best to go about explaining particular concepts and the solution to particular questions, especially those questions which are pushing the limits of the course material.

Self-generated feedback is not something I have formally undertaken much of. That is not to say I have *never* completed any self-evaluation; on the contrary: since I took three different tutorial groups for the same module I would often mentally reflect on what went well or poorly in the first tutorial of the week, and use this to inform the later ones. I could make note of which explanations and examples were well-received and which needed re-thinking. I could observe which topics the group generally struggled with, and plan appropriate explanations to use with the next group. Because of this, it always felt to me that the last tutorial of the week was significantly more polished than the first tutorial.

I realise, however, that just making a mental note of what went well or poorly has not afforded me a complete picture of my performance with the three tutorial groups, or of each group's idiosyncrasies. If I had had the foresight to make a written note of my self-evaluations I would had a record of what techniques worked with each tutorial group and would be able to better plan my future explanations, and adapt my teaching style. When I take on another set of tutorials in the coming year, I will make *formalising* and *recording* the results of my self-evaluations and self-reflections a higher priority.

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A Peer Review

Peer observation of Michaelmas 2013 tutorial by a member of teaching staff:

> Your tutorial seemed to go well, the students were generally engaged. Covering problems with the homework at the start of the tutorial was a good idea, students could use this with the tutorial questions. Maybe you spent too long on just the homework. Getting the students working through the problems in groups at the boards was interesting, but it seems to fit well with the style of the analysis questions, I might give it a try in my tutorials. You were able to confidently answer any questions the students asked, and gave good feedback on their solutions. Having longer at the end of the tutorial to recap the questions could be useful.

Peer observation of Easter Revision 2014 tutorial by a fellow PhD student:

- Tutorial started on time.
- Had enough pens, etc, for the whiteboards. No problems with pens running out or struggling to clean the boards.
- Students were comfortable asking questions and discussing problems.
- Asked students for any particular topics or problems they wanted to cover.
- Gave an impressive and detailed explanation of uniform convergence with little rehearsal when asked, but maybe it was too long for such a small part of the course.
- Reminded students they can get in contact via email or coming to the office.
- Tutorial overran slightly.

B Student Questionnaire

The student questionnaire has questions about: content and delivery, course delivery and communication, work and feedback, resources, tutorials, and the course overall. Below I reproduce the responses to the questions on work and feedback, and tutorials. These are the questions relevant to my teaching practice.

Michaelmas 2013

Q: The work set is appropriate in terms of quantity, difficulty and relevance; feedback/marking is timely, accurate and helpful.[3]

Strongly agree	Agree	Neither	Disagree	Strongly disagree	Average Score
7	12	2	1	0	78%[4]

- I would prefer to have a set of solutions online that I could look at when I need help doing similar questions to ones covered previously in a tutorial.
- Marking could be improved abit [sic] to give abit [sic] more help on the solution to incorrect answers.
- The work is always proptly [sic] returned and well marked. Tutorials ([tutor]) are always useful.
- There is just the right amount of tutorial and homework questions which means that I'm tested in all of the relevant areas but not given ridiculous amounts of work to do.

[3] I am not responsible for the work set. Only responses concerning 'feedback/marking is timely, accurate and helpful' are relevant to my teaching practice.

[4] Value as computed by the DUO enterprise survey tool.

Q: The tutorials help me to understand the material and solve problems, and have helped to develop my communication skills, confidence and interest in mathematics.

Strongly agree	Agree	Neither	Disagree	Strongly disagree	Average Score
7	11	3	1	0	77%

- [Tutor] needs to be a bit more directive in what he wants us to do, when he wants us to work in groups at the boards etc. We need more time to go over the solutions at the end as a group.
- Analysis tutorials with [tutor] are the highlight of my week.
- Dislike the setup of the tutorial as I feel little guidance is given when I am finding a question or concept tricky and thereofe [sic] it is difficult to make progress.
- I really enjoy working on the tutorial questions in small groups and having the tutor coming round at various points to point us in the right direction. I also feel that going over the homework at the start of the tutorial is really useful. However, it would be more helpful if he spent longer going over the tutorial questions at the end as I sometimes feel like I still don't understand some of the harder questions.
- I think it would be better to recap the topic during the tutorial and to then discuss the questions with more confidence afterwards. I think it would also be helpful to hand the homework in after the tutorial so it gives more of an opportunity to cover all the work during the lectures and then ask questions in the tutorials which would allow time for the homework to be done better.

Epiphany 2014

Q: The work set is appropriate in terms of quantity, difficulty and relevance; feedback/marking is timely, accurate and helpful.

Strongly agree	Agree	Neither	Disagree	Strongly disagree	Average Score
7	11	1	0	0	83%

- The questions don't seem 'hard' but there are so many opportunities to loose marks by missing out part of a proof etc. It generally seems relevant to the course but sometimes the questions are focussed on things from a few weeks back or something we haven't covered yet (eg. Mean Limit Theorem) - it would be better if the homework reflected the work we were doing at the time. The feedback is good and useful.
- The work is pitched at just the right level. Sometimes I feel that it isn't sufficiently covered in lectures though and you have to rely on tutorials to some extent in order to know how to tackle problems.

Q: The tutorials help me to understand the material and solve problems, and have helped to develop my communication skills, confidence and interest in mathematics.

Strongly agree	Agree	Neither	Disagree	Strongly disagree	Average Score
9	8	1	0	1	82%

- I appreciate that the tutor goes through the homework at the start and helps us to understand it and discussing the problems in small groups is useful.
- I enjoy the tutorials. I like how the tutor goes through any problems with the homework at the start of the tutorial as it helps me to get more from the tutorial questions.

- I like the summary sheets my tutor has given me on some of the more challenging material, this helped to clarify it.
- These tutorials are really really good, they aid in understanding and are also enjoybale [sic] in working through the problems.
- Tutor always very helpful and gives good feedback.

C Email Feedback from Students

Below I reproduce some responses from students I have helped via email

- Oh yes I forgot that you use the integral test for things like that, but looking back over it that helps a lot, thank you!

- I've only just got round to having a look at your solution. I understand it now! Thank you so much for your help - I feel like I get the whole epsilon N thing much better now in general.

  Thanks again!

- Thankyou [sic] very much, I understand both of the questions a lot better now. Thanks for such a detailed and comprehensive answer to my questions.

  Thanks again[.]

D Limits Handout

Below I reproduce a handout I prepared for my students about proving limits via the ε-δ definition.

ϵ-δ Proofs

See [tutor's module homepage] for updates.

When you first meet them, ϵ-δ proofs are conceptually quite difficult. It usually takes a lot of time and effort thinking over the ideas before the concept finally 'clicks', makes sense and seems natural.

Firstly recall the definition:

Definition (ϵ-δ limit definition)**.** We say $\lim_{x\to c} f(x) = L$ if:

$$\text{Given any real } \epsilon > 0, \text{ we can find some real } \delta > 0 \text{ such that}$$
$$0 < |x-c| < \delta \implies |f(x) - L| < \epsilon$$

⟦ In this definition L and c are real numbers (or complex numbers), in particular L and c are finite. Limits as $x \to \infty$, or limits which equal infinity require different definitions. ⟧

So let's say that you claim some function $f(x)$ has limit L at c. Then if I pick any ϵ, you should be able to give me the corresponding δ: I tell you how close to L I want the output of f to be (within ϵ), then you me how close to c I need to look (within δ).

1 Numerical Example

Say we're looking at $f(x) = 3x - 1$. You claim $f(x)$ has limit $L = 5$ at $c = 2$.

I want $\epsilon = 1$. We find $|3x - 1 - 5| < 1 \iff |3(x-2)| < 1 \iff |x - 2| < 1/3$. So you can tell me to take $\delta = 1/3$. That is, if I plug in any x from the interval $(2 - 1/3, 2 + 1/3)$, the output I get from $f(x)$ is in the interval $(5 - 1, 5 + 1)$.

Now I want $\epsilon = 1/7$. Similarly we find $|3x - 1 - 5| < 1/7 \iff |3(x-2)| < 1/7 \iff |x - 2| < 1/21$. So you will tell me to take $\delta = 1/21$.

We can do this over and over again (and not just with rational numbers), and make a table like

Given ϵ	Found δ
1	1/3
1/7	1/21
6/101	2/101
0.60335 . . .	0.20111 . . .
0.035516 . . .	0.011838 . . .

When dealing with a particular function, the δ you give back to me can only possibly depend on the ϵ I give to you. That is, δ is a function of ϵ only. The expression you finally write down for $\delta = \cdots$ can only contain ϵ, it can't contain x or anything else.

2 Example ϵ-δ Proofs

In this section don't worry (too much) about why I choose a particular values for δ in the proof. Just concentrate on the structure of the proof itself, unencumbered by the working which you would have to do beforehand to find the value to use.

This will make it clearer what is the ϵ-δ proof, and what is the working out we need to do alongside.

Problem: Give an ϵ-δ proof that $\lim_{x\to 2}(3x-1) = 5$.

Solution: Given $\epsilon > 0$, let $\delta = \epsilon/3$. If $|x - 2| < \delta$ then

$$|3x - 1 - 5| = |3(x-2)| = 3\underbrace{|x-2|}_{<\delta} < 3\delta = 3 \cdot \epsilon/3 = \epsilon\,.$$

So the ϵ-δ definition holds, and $\lim_{x\to 2}(3x-1) = 5$, as required. □

Problem: Give an ϵ-δ proof that $\lim_{x\to 4} x^2 = 16$.

Solution: Given $\epsilon > 0$, let $\delta = \min\{1, \epsilon/9\}$. Note this means $\delta < 1$ and $\delta < \epsilon/9$.
If $|x - 4| < \delta$ then

$$|x-4| < 1 \implies -1 < x - 4 < 1 \implies 7 < x + 4 < 9 \implies |x+4| < 9\,,$$

and

$$\left|x^2 - 16\right| = \underbrace{|x+4|}_{<9}|x-4| < 9\underbrace{|x-4|}_{<\delta} < 9\delta < 9 \cdot \epsilon/9 = \epsilon\,.$$

So the ϵ-δ definition holds, and $\lim_{x\to 4} x^2 = 16$, as required. □

Problem: Given an ϵ-δ proof that $\lim_{x\to 3} \frac{x+3}{(x+1)(x-2)} = \frac{3}{2}$.

Solution: Given $\epsilon > 0$, let $\delta = \min\{\frac{1}{2}, \frac{\epsilon}{5}\}$. Note this means $\delta < \frac{1}{2}$ and $\delta < \frac{\epsilon}{5}$.
Firstly if $|x - 3| < \frac{1}{2}$, then

$$-\tfrac{1}{2} < x - 3 < \tfrac{1}{2} \implies \tfrac{5}{2} < x < \tfrac{7}{2}\,.$$

So

$$\tfrac{7}{2} < x + 1 < \tfrac{9}{2} \implies \tfrac{1}{|x+1|} = \tfrac{1}{x+1} < \tfrac{2}{7}\,,$$

and

$$\tfrac{1}{2} < x - 2 < \tfrac{3}{2} \implies \tfrac{1}{|x-2|} = \tfrac{1}{x-2} < 2\,,$$

and

$$\tfrac{23}{2} < 3x + 4 < \tfrac{29}{2} \implies |3x+4| < \tfrac{29}{2}$$

Now if $|x - 3| < \delta$, then $|x - 3| < \frac{1}{2}$, and

$$\left|\frac{x+3}{(x+1)(x-2)} - \frac{3}{2}\right| = \frac{|3x+4|\,|x-3|}{2\,|x+1|\,|x-2|} < \frac{29}{2} \cdot \delta \cdot \frac{1}{2} \cdot \frac{2}{7} \cdot 2 = \frac{29}{7}\delta < \frac{29}{7}\frac{\epsilon}{5} = \frac{29}{35}\epsilon < \epsilon$$

So the ϵ-δ definition holds, and $\lim_{x\to 3} \frac{x+3}{(x+1)(x-2)} = \frac{3}{2}$, as required. □

3 Writing Your Own ϵ-δ Proofs

Now you should be wondering how I came up with the particular values for δ in the above proofs? This time I'll include the working out needed beforehand.

Problem: Give an ϵ-δ proof that $\lim_{x\to 3} x^2 + 2x - 1 = 14$.

Solution: Firstly:

$$|f(x) - L| = \left|x^2 + 2x - 1 - 14\right| = \left|x^2 + 2x - 15\right| = |x-3|\,|x+5|$$

Now we have a term $|x - 3|$ which we know will be $< \delta$ in the proof. But how can we deal with the other term $|x + 5|$? Can we say anything about it? Well, we have: $|x - 3| < \delta \implies -\delta < x - 3 < \delta \implies 8 - \delta < x + 5 < 8 + \delta$, so $|x + 5| < 8 + \delta$.

We could make this work, but why deal with arbitrary δ, and quadratic equations in δ? The limit only cares what goes on close to $x = 3$, so we can take (arbitrarily) $\delta \le 1$. If $\delta = 1$ works for $\epsilon = 1/2$, then it also works for $\epsilon = 50$; if it gives $|f(x) - 14| < 1/2$, then obviously we also have $|f(x) - 14| < 50$ since $1/2 < 50$.

If we assume $\delta \le 1$, the we have $|x + 5| < 9$, so we get:

$$|x-3|\,|x+5| < 9\delta\,.$$

We can make $|x - 3|\,|x + 5| < \epsilon$ by making $9\delta \le \epsilon$, so by taking $\delta \le \epsilon/9$.

So we must have $\delta \le 1$, and $\le \epsilon/9$, i.e. $\delta \le \min\{1, \epsilon/9\}$. Therefore let's take $\delta = \min\{1, \epsilon/9\}$. We've now finished the working out, and so we can start the proof proper. The above does work as a proof, as long as you make sure all the implications are in the right direction, but it's good to see directly this works:

Given $\epsilon > 0$, let $\delta = \min\{1, \epsilon/9\}$. Note $\delta \le 1$ and $\le \epsilon/9$. Then:

$$|x-3| < \delta \implies |x-3| < 1 \implies |x+5| < 9\,.$$

So, if $|x - 3| < \delta$, then:

$$|f(x) - L| = \underbrace{|x+5|}_{<9}\underbrace{|x-3|}_{<\delta} < 9\delta < 9 \cdot \epsilon/9 = \epsilon\,.$$

So the ϵ-δ definition holds, and $\lim_{x\to 3} \frac{x+3}{(x+1)(x-2)} = \frac{3}{2}$, as required. □

Problem: Give an ϵ-δ proof that $\lim_{x\to 4} \frac{1}{x-3} = 1$.

Solution: Firstly:

$$|f(x) - L| = \left|\frac{1}{x-3} - 1\right| = \frac{|x-4|}{|x-3|}\,.$$

We have the $|x - 4|$ bit which we know is $< \delta$. Can we find an upper bound on $\frac{1}{|x-3|}$ after restricting δ a bit? Let's assume $\delta \le 1$. Then $|x - 4| < \delta \implies |x - 4| < 1 \implies -1 < x - 4 < 1 \implies 0 < x - 3 < 2$. And all this says is $\frac{1}{|x-3|} = \frac{1}{|x-3|} > 1/2$, which is no good.

To fix this we need to restrict δ even more. The problem is the vertical asymptote $x = 3$ which is only 1 unit away from the limit point at $x = 4$. Taking anything < 1 means we don't reach all the way to the asymptote, and so there are no problems.

So assume $\delta \le 1/2$. Then $|x - 4| < 1/2 \implies 1/2 < x - 3 < 3/2 \implies \frac{1}{|x-3|} < 2$. So we get

$$|f(x) - L| = \frac{|x-4|}{|x-3|} < 2\delta\,,$$

and we can make it $< \epsilon$ by taking $\delta \le \epsilon/2$.

So taking $\delta = \min\{1/2, \epsilon/2\}$ gives us the ϵ-δ proof. □

4 General Strategy for Rational Functions

Suppose we have to give an ϵ-δ proof that $\lim_{x\to c} \frac{a(x)}{b(x)} = L$. Do the following:

Step 1: Compute $\left|\frac{a(x)}{b(x)} - L\right|$

Step 2: Write it in the form $|x - c|\,\frac{|p(x)|}{|q(x)|}$, where $p(x)$ and $q(x)$ are polynomials, and $q(c) \ne 0$. You *will* be able to do this. ⟦ First cancel all factors of $(x - c)$ on top and bottom, so you can evaluate the limit by substituting in $x = c$. There can't be a factor of $(x - c)$ on the bottom since the limit exits, so $q(c) \ne 0$. And there has to be a factor of $(x - c)$ on the top since at $x = c$, we get L. ⟧

Step 3: Find the nearest asymptote (root of $q(x)$) to the point $x = c$. Pick any D less than this distance. Assume $\delta \le D$.

Step 4: Use $|x - c| < D$ to say $c - D < x < c + D$, and use this to find an upper bound P on $p(x)$. So get $|x - c| < \delta \implies |p(x)| < P$.

Step 5: Also use this to find a lower bound Q on $q(x)$. So get $|x - c| < \delta \implies \frac{1}{|q(x)|} < \frac{1}{Q}$.

Step 6: Using these we get

$$\left|\frac{a(x)}{b(x)} - L\right| = |x-c|\,\frac{|p(x)|}{|q(x)|} < \delta\frac{P}{Q}\,,$$

so we can make it $< \epsilon$, by taking $\delta < \frac{Q}{P}\epsilon$ as well.

Take $\delta = \min\{D, \frac{Q}{P}\epsilon\}$, then $|x - c| < \delta \implies \left|\frac{a(x)}{b(x)} - L\right| < \epsilon$. Hence we get an ϵ-δ proof that $\lim_{x\to c} \frac{a(x)}{b(x)} = L$.