

## NUMBERS! - PROBLEM SHEET 2

- (1) Check that the list of 20 surreal numbers formed from subsets of  $\{-1, 0, 1\}$  is complete. How long is the list formed from subsets of  $\{-2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2\}$  on Day 3?
- (2) Complete the proof to show  $\{0 \mid 1\} < 1$ .
- (3) Check some of the Day 2 (in)equalities directly. Check, say,  $\{-1 \mid 0\} < \{-1 \mid 1\}$ . But you *may not* use that  $\{-1 \mid 1\} = 0$  to say  $-1 \not\leq \{-1 \mid 1\}$ , since we have not yet proved that  $x = y$  and  $y \not\leq z$  implies  $x \not\leq z$ !

We will prove the following Proposition soon.

**Proposition.** *Let  $x = \{X_L \mid X_R\}$  be a surreal number. If  $y \not\leq x$ , then  $x = \{y, X_L \mid X_R\}$ . Also, if  $y \leq x$ , then  $x = \{X_L \mid X_R, y\}$ .*

- (4) Use this proposition to justify the equalities for the Day 2 surreal numbers.
- (5) Assume it known that the Day 2 surreal numbers are totally ordered:  $-2 < -1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1 < 2$ . Convince yourself that only the largest element of  $X_L$ , and the smallest element of  $X_R$  matter when checking  $=$ . Now try to list the surreal numbers on day 3.
- (6) You should have found the number  $\{0 \mid \frac{1}{2}\}$  on Day 3. What do you think its value should be? (We will determine and justify its value later.)
- (7) What happens on Day 4? How many new numbers? Can you guess the pattern?